AS [89 marks]

1. [Maximum mark: 6]

The first three terms of an arithmetic sequence are u_1, $5u_1 - 8$ and $3u_1 + 8$.

(a) Show that $u_1 = 4$. [2]

(b) Prove that the sum of the first n terms of this arithmetic sequence is a square number. [4]

2. [Maximum mark: 6]

The 1 st and 5 th terms of an arithmetic sequence are 36 and 12 respectively.

(a) Find the 13 th term of this arithmetic sequence. [4]

The sum of the first n terms of this arithmetic sequence is zero.

(b) Find the value of n. [2]

3. [Maximum mark: 5]

The 7 th term of an arithmetic sequence is 6.

The sum of the 6 th term and the 12 th term is 24.

Find the first term and the common difference. [5]

4. [Maximum mark: 5]

Consider the sequence $\{u_n\}$, with $n\text{ th}$ term given by u_n. The first three terms are

$u_1 = k - 5,\ u_2 = 3 - 2k$ and $u_3 = 5k + 3$, where $k \in \mathbb{R}$.

(a) Consider the case when $\{u_n\}$ is arithmetic.

(a.i) Find the value of k. [3]

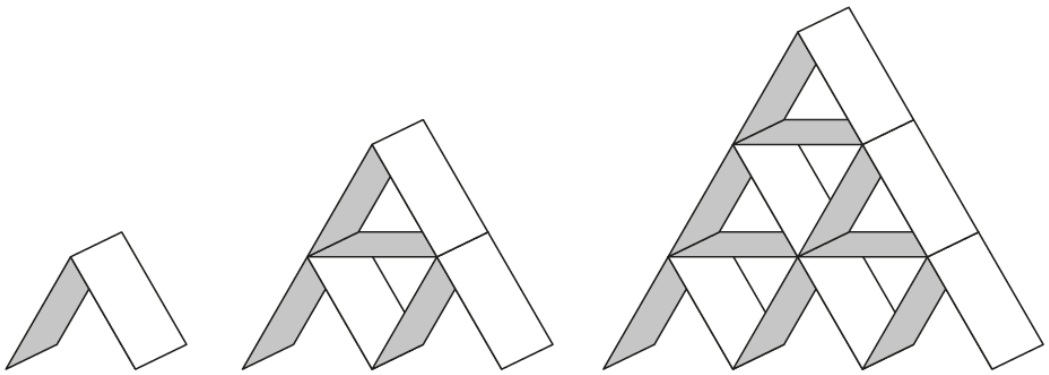
(a.ii) Hence, or otherwise, find u_3. [2]

5. [Maximum mark: 16]

Rectangular playing cards are stacked in the shape of a pyramid with n rows, where $n \geq 1$.

Some cards are placed horizontally and some cards are stacked at an angle of 60° to the horizontal.

The following diagrams represent pyramid stacks for $n = 1, n = 2$ and $n = 3$.



$n = 1$ $n = 2$ $n = 3$

Let t_n represent the number of cards used to create a pyramid stack with n rows.

(a) Write down t_3. [1]

(b) Find t_4. [2]

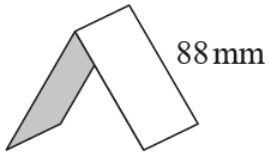
(c) Show that $t_n = \frac{n(3n+1)}{2}$. [3]

There are 52 cards in a full pack of playing cards.

(d) A complete pyramid stack is created using playing cards taken from 14 full packs. Find the maximum number of rows in this stack. [3]

(e) A complete pyramid stack is created using playing cards taken from full packs with no cards left over. Find the minimum number of rows in this stack. [2]

The long edge of each playing card measures $88\ \text{mm}$ as illustrated in the following diagram.



(f) Find the minimum number of cards needed to create a complete pyramid stack with a vertical height of more than 2 metres. The thickness of the cards may be ignored. [5]

6. [Maximum mark: 6]

For a particular arithmetic sequence, $u_{10} = 14$ and $S_{25} = 200$.

Find the value of k such that $u_k = 0$. [6]

7. [Maximum mark: 6]

For a particular arithmetic sequence, $u_{10} = 16$ and $S_{25} = 100$.

Find the value of k such that $u_k = 0$. [6]

8. [Maximum mark: 4]

The second term of an arithmetic sequence is 10 and the fourth term is 22.

(a) Find the value of the common difference. [2]

(b) Find an expression for u_n, the n th term. [2]

9. [Maximum mark: 7]

The sum of the first n terms of an arithmetic sequence is given by $S_n = pn^2 - qn$, where p and q are positive constants.

It is given that $S_5 = 65$ and $S_6 = 96$.

(a) Find the value of p and the value of q. [5]

(b) Find the value of u_6. [2]

10. [Maximum mark: 11]

Consider the arithmetic sequence $u_1, u_2, u_3, \ldots$.

The sum of the first n terms of this sequence is given by $S_n = n^2 + 4n$.

(a.i) Find the sum of the first five terms. [2]

(a.ii) Given that $S_6 = 60$, find u_6. [2]

(b) Find u_1. [2]

(c) Hence or otherwise, write an expression for u_n in terms of n. [3]

(d) Given that $v_{99} < 0$, find v_5. [2]

11. [Maximum mark: 2]
Consider an arithmetic sequence with $u_1 = 0.6$ and $u_4 = 0.15$.

(a) Find the common difference, d. [2]

12. [Maximum mark: 5]
The n^{th} term of an arithmetic sequence is given by $u_n = 15 - 3n$.

(a) State the value of the first term, u_1. [1]

(b) Given that the n^{th} term of this sequence is -33, find the value of n. [2]

(c) Find the common difference, d. [2]

13. [Maximum mark: 5]
Consider an arithmetic sequence where $u_8 = S_8 = 8$. Find the value of the first term, u_1, and the value of the common difference, d. [5]

14. [Maximum mark: 5]
An arithmetic sequence has first term 60 and common difference -2.5.

(a) Given that the kth term of the sequence is zero, find the value of k. [2]

(b) Let S_n denote the sum of the first n terms of the sequence.

Find the maximum value of S_n. [3]